

CREDIT CONTRACTS WITH PERFECT INFORMATION



- Introduction: valuation of contingent claims
- Valuing firm and debt: the option theory approach
- Valuation of loan guarantees



1-INTRODUCTION: VALUATION OF CONTINGENT CLAIMS

- A general method to value derivatives and contingent claims
- Developed by Black and Scholes (1973) and Merton (1971 to 1974)
- Applications of contingent claim analysis in corporate finance is very powerful



1-INTRODUCTION: VALUATION OF CONTINGENT CLAIMS

- Hypothesis (Black and Scholes, 1973; Merton, 1974)
 - Assets perfectly divisible, no taxes, no transaction costs
 - « Price takers » investors
 - There is a riskless asset which pays r per unit of time
 - The value of the firm is independent from the structure of its liabilities
 - Term structure of interest rates is flat; the value of a zero-coupon bond which pays 1 euro in date τ is $p(\tau)=\exp(-r \tau)$
 - The value V of the assets of the firm follows
 - $dV=(\alpha V-C)dt+\sigma Vdz$
 - α =instant return of the assets per unit of time
 - C =instant cash-flow paid by the firm to shareholders, creditors
 - σ =standard deviation of the asset return per unit of time
 - dz is a standard Gauss-Wiener process



1-INTRODUCTION: VALUATION OF CONTINGENT CLAIMS

- Suppose there exists an asset whose value Y can be written as a function of V and time t , $f(V,t)$; then:

$$dY = (\alpha_y Y - C_y) dt + \sigma_y Y dz_y$$

Variables have the same meaning as before

- By Ito's lemma, we have:

$$dY = F_V dV + \frac{1}{2} F_{VV} (dV)^2 + F_t dt = \left[\frac{1}{2} \sigma^2 V^2 F_{VV} + (\alpha V - C) F_V + F_t \right] dt + \sigma V F_V dz$$



1-INTRODUCTION: VALUATION OF CONTINGENT CLAIMS

- Comparing the 2 previous equations leads to:

$$\alpha_y Y = \alpha_y F = \left[\frac{1}{2} \sigma^2 V^2 F_{VV} + (\alpha V - C) F_V + F_t + C_y \right]$$

$$\sigma_y Y = \sigma_y F = \sigma V F_V$$

$$dz_y = dz$$



1-INTRODUCTION: VALUATION OF CONTINGENT CLAIMS

- With Merton, consider a portfolio invested in
 - The firm for an amount W_1
 - The claim on that firm (Y) for W_2
 - The riskless asset for W_3
 - Such that $W_1 + W_2 + W_3 = 0$



1-INTRODUCTION: VALUATION OF CONTINGENT CLAIMS

- Such a strategy generates an instant return dx :

$$dx = W_1 \frac{dV + Cdt}{V} + W_2 \frac{dY + C_y dt}{Y} + W_3 r dt$$

- Using the dynamics dV and dY , we get:

$$dx = \left[W_1(\alpha - r) + W_2(\alpha_y - r) \right] dt + (W_1\sigma + W_2\sigma_y) dz$$



1-INTRODUCTION: VALUATION OF CONTINGENT CLAIMS

- Suppose a portfolio whose return is non stochastic (coef of dz is nil), then its return is certain and equals 0 because the initial investment is also nil; so:

$$W_1\sigma + W_2\sigma_y = 0$$

$$W_1(\alpha - r) + W_2(\alpha_y - r) = 0$$



1-INTRODUCTION: VALUATION OF CONTINGENT CLAIMS

- A non trivial solution imposes that:

$$\frac{\alpha - r}{\sigma} = \frac{\alpha_y - r}{\sigma_y}$$

- Using equations in slide 5, we get:

$$\frac{\alpha - r}{\sigma} = \frac{\frac{1}{2}\sigma^2 V^2 F_{VV} + (\alpha V - C)F_V + F_t + C_y - rF}{\sigma V F_V}$$



1-INTRODUCTION: VALUATION OF CONTINGENT CLAIMS

- By rearranging the terms and simplifying, Merton gets finally:

$$0 = \frac{1}{2} \sigma^2 V^2 F_{VV} + (rV - C) F_V - rF + F_t + C_y$$

- This equation must be satisfied by every claim whose value can be written as a function of time and of the value of the firm
- The solution of that equation requires that initial and boundary conditions must be defined
- Such boundary conditions distinguish an asset from another



2-VALUING FIRM AND DEBT:THE OPTION THEORY APPROACH

- Some more hypothesis
 - A firm is financed by equity and a single class of debt
 - Debt pays a unique amount B at maturity T
 - No coupons
 - Firm doesn't pay any dividends



2-VALUING FIRM AND DEBT:THE OPTION THEORY APPROACH

- With such hypothesis, the general valuation formula applied to the debt simplifies in:

$$0 = \frac{1}{2} \sigma^2 V^2 F_{VV} + rVF_V - rF - F_\tau$$

- Where $\tau = T - t$ is time before maturity



2-VALUING FIRM AND DEBT:THE OPTION THEORY APPROACH

- Boundary conditions are the following:

$$F(0, \tau) = 0$$

$$F(V, \tau) \leq V$$

$$F(V, 0) = \text{Min}(V, B)$$

- Valuation formula in slide 12 and the boundary conditions are sufficient to value risky debt of that firm



2-VALUING FIRM AND DEBT:THE OPTION THEORY APPROACH

- Following Merton (1974), a closed-form valuation formula can be obtained
- If we call $f(V, \tau)$ the value of equity, then we have:
 - $F(V, \tau) + f(V, \tau) = V$
$$0 = \frac{1}{2} \sigma^2 V^2 f_{VV} + rVf_V - rf - f_\tau$$
 - With boundary condition $f(V, 0) = \text{Max}(0, V - B)$



2-VALUING FIRM AND DEBT:THE OPTION THEORY APPROACH

- Equations in slide 14 show that the value of equity is equivalent to the value of a call option
 - Strike is payment B to creditors
 - Exercise date is debt maturity
 - Underlying asset is the value of the firm assets
- Then, we can apply the Black-Scholes option formula to value the firm's equity



2-VALUING FIRM AND DEBT: THE OPTION THEORY APPROACH

- We get:

$$f(V, \tau) = V.N(d_1) - B.exp(-r\tau).N(d_2)$$

$$d_1 = \frac{\left[\text{Log}(V/B) + \left(r + \frac{\sigma^2}{2} \right) \tau \right]}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$



2-VALUING FIRM AND DEBT:THE OPTION THEORY APPROACH

- Following Merton (1974), the value of the risky debt is now:

$$F(V, \tau) = V.N(-d_1) + B.exp(-r\tau).N(d_2)$$

- Divide this expression by $B.exp(-r\tau)$
- Define $d = \frac{B.exp(-r\tau)}{V}$ as the quasi-leverage
- We get $\frac{F(V, \tau)}{B.exp(-r\tau)} = \frac{N(-d_1)}{d} + N(d_2)$



2-VALUING FIRM AND DEBT:THE OPTION THEORY APPROACH

- Define R as the yield to maturity:

$$F(V, \tau) = B \cdot \exp(-R\tau)$$

- Finally, the risk premium becomes:

$$R - r = -\frac{1}{\tau} \text{Log} \left[N(d_2) + \frac{N(-d_1)}{d} \right]$$

- The risk premium is a positive function of:
 - The business risk σ
 - The quasi-leverage d



2-VALUING FIRM AND DEBT: THE OPTION THEORY APPROACH

- An example
 - Firm's assets value
 - 1000 \longrightarrow 1200 with prob 0,5
 - 1000 \searrow 1000 with prob 0,5
 - Debt promises a payment of 1050 in one period
 - The riskless rate is 3%
 - Compute the value of equity, debt and risk premium

2-VALUING FIRM AND DEBT: THE OPTION THEORY APPROACH

- Assets value: 1000 $\begin{array}{l} \longrightarrow 1200 \\ \searrow 1000 \end{array}$
- Equity: S $\begin{array}{l} \longrightarrow 150 \\ \searrow 0 \end{array}$
- Consider a portfolio invested in assets (m) and equity (n); such a strategy pays:
 - 1200m+150n in the good state
 - 1000m in the bad state



2-VALUING FIRM AND DEBT:THE OPTION THEORY APPROACH

- You can choose (m,n) such that the portfolio is riskless:
 - $1200m+150n=1000m$
 - For example, $m=3; n=-4$
 - The portfolio includes 3 assets and is financed by selling 4 call on these assets
- this strategy ensures a sure return:
 - $1000m=3000$



2-VALUING FIRM AND DEBT:THE OPTION THEORY APPROACH

- The non arbitrage condition imposes:
 - $3000-4S=3000/(1+3\%)$
 - $S=21,84$
- Debt value is:
 - $1000-21,84=978,16$
- Yield to maturity is:
 - $(1050/978,16)-1=7,34\%$
- Risk premium is:
 - $7,34\%-3\%=4,34\%$

3-VALUATION OF LOAN GUARANTEES

- How to value a guaranteed debt?
 - Jones and Mason (1980) observe that:
 - value a guaranteed debt = value of a non guaranteed debt + value of a guarantee
 - $F^*(V,t) = F(V,t) + G(V,t)$
 - F and G are contingent claims
 - F verifies $0 = \frac{1}{2} \sigma^2 V^2 F_{VV} + rVF_V - rF - F_\tau$
 - G verifies $0 = \frac{1}{2} \sigma^2 V^2 G_{VV} + rVG_V - rG - G_\tau$

3-VALUATION OF LOAN GUARANTEES

- These 2 equations must be completed by boundary conditions which in turn depend of the level of the guarantee
- The case of a fully guaranteed debt
 - $G(V,0)=\text{Max}(0,B-V)$
 - $G(0,\tau)=B$
 - $G(\infty,\tau)=0$



3-VALUATION OF LOAN GUARANTEES

- The case of a junior debt
 - A senior debt is reimbursed prioritarily/junior debt
 - Junior debt=subordinated debt=mezzanine debt
 - Senior debt is cheaper than junior debt
 - Ex of senior debt=bank debt
 - Ex of junior debt=bonds



3-VALUATION OF LOAN GUARANTEES

- Suppose a firm has 2 classes of debt
 - Junior debt promises to pay c' by unit of time, and B' at maturity
 - Senior debt promises to pay c by unit of time, and B at maturity
 - Both debt have same maturity
 - Firm pays dividends d by unit of time
 - Senior debt is unguaranteed, junior debt is guaranteed

3-VALUATION OF LOAN GUARANTEES

- How to value the junior debt?
 - The value of the unguaranteed junior debt obeys to the following equation

$$0 = \frac{1}{2} \sigma^2 V^2 F_{VV} + (rV - C) F_V - rF - F_\tau + c'$$

With $C = c + c' + d$

- At maturity, unguaranteed junior debt receives the residual value of firm assets after senior creditors have been paid, up to a maximum of B' .
- $F(V, 0) = \text{Min}[B', \text{Max}(0, V - B)]$

3-VALUATION OF LOAN GUARANTEES

- The value G of the guarantee satisfies the following equation:

$$0 = \frac{1}{2} \sigma^2 V^2 G_{VV} + (rV - C) G_V - rG - G_\tau$$

- With $C = c + c' + d$ and the boundary condition:
 - $G(V, 0) = \text{Min}[B', \text{Max}(0, B + B' - V)]$



SUMMARY

- Contingent claim analysis offers a general framework to value stocks and various classes of debts
- The risk premium depends on the business risk and the quasi-leverage
- Guarantees can also be valued as contingent claims



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